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Absence of Debye Screening in the Quantum Coulomb System

David C. Brydges[1] **and Georg Keller**[1]

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We present an approximation to the quantum Coulomb plasma at equilibrium which captures the power-law violations of Debye screening which have been reported in recent papers. The objectives are (1) to produce a simpler model which we will study in forthcoming papers, and (2) to develop a strategy by which the absence of screening can be proven for the low-density quantum Coulomb plasma itself.

KEY WORDS: Screening; plasma; Coulomb; van der Waals.

1. THE CLASSICAL COULOMB GAS

The partition function for a (charge-symmetric) classical Coulomb gas in three dimensions is

$$Z = \sum \frac{z^N}{N!} \int d^N p \, d^N \xi \, e^{-\beta H} \tag{1}$$

where $\xi = (x, e)$ and $d\xi$ unites an integral over $x \in$ container with a sum over charges $e = \pm 1$. We have

$$H = \sum \frac{p_i^2}{2m} + \frac{1}{2} \int \rho v_l \rho + i \int \phi_{\text{ext}} \rho \tag{2}$$

where we define the charge density observable by

$$\rho(x) = \sum e_i \delta(x - x_i) \tag{3}$$

[1] Department of Mathematics, University of Virginia, Charlottesville, Virginia 22903.

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so that

$$\int \rho v_l \rho = \sum e_i e_j v_l(x_i - x_j) \tag{4}$$

$i\phi_{\text{ext}}$ is an external field. We put in the strange factor i because it will lead to simpler expressions when we explain the sine-Gordon transformation. $v_l(x-y) = $ "$1/r$." We have put the quotes around the $1/r$ because it is necessary to place a cutoff on the singularity of the Coulomb potential at short distances in order to have a stable interaction. This cutoff will be characterized by a length l. In particular, $v_l(0) = 1/l$. Having enforced a cutoff, the self-energies of the particles are finite and we have included them in the interaction energy. The natural choice for this length l is the thermal wavelength, which is the size of the typical one-particle wavefunction in a corresponding quantum ideal gas

$$l = \left(\frac{\beta \hbar^2}{m}\right)^{1/2} \tag{5}$$

since it is the Pauli exclusion principle and quantum mechanics that give rise to a stable system which we are approximating classically.

The other lengths which naturally arise are β and the Debye length

$$l_{\text{D}} = \frac{1}{(2z\beta)^{1/2}} \tag{6}$$

where

$$z = \tilde{z} \int dp\, e^{-(\beta/2m)p^2} e^{-\beta/2l} \tag{7}$$

z is the physical activity in the sense that the expectation of the density of particles is asymptotic to $2z$ as $z \to 0$. The factor $e^{-\beta/l}$ accounts for the inclusion of the self-energies in the interaction.

For this system the following theorem has been proved.[4,5,9,10]

Theorem 1. For

$$zl^3 \ll e^{-\beta/2l}, \qquad zl_{\text{D}}^3 \gg 1 \tag{8}$$

all charge–charge correlations decay exponentially, i.e., there are constants C_1 and $L > 0$ such that

$$|\langle \rho(x)\, \rho(y) \rangle| \leqslant C_1 e^{-|x-y|/L} \tag{9}$$

and higher truncated charge correlations decay exponentially as the length of the shortest tree on the positions of the observables.

Also $L \simeq l_{\text{D}}$ when zl^3 and zl_{D}^3 are as in the theorem.

2. DISCUSSION

In ref. 5, p. 428, it was claimed that screening of observables in the sense of exponential decay as in the theorem above will not hold for the quantum plasma. The argument was strengthened by some lower bounds (but only on time-dependent observables) given by Brydges and Seiler.[3] Since then Alastuey and Martin[1,2] have made detailed calculations which state that within perturbation theory (the Wigner–Kirkwood expansion) screening is destroyed by effects due to diagrams with power-law decay at order $\hbar^4$ and higher. They show, for example, that for NaCl ions in water at room temperature there will be screening out to about 60 Debye lengths, at which point there is a crossover to a power-law tail. According to their analysis the typical power law is r^{-6}, but it can be higher, depending on the correlation and the system. This violation of screening has nothing to do with statistics. It is similar in mechanism to van der Waals forces, but it occurs[2] even for one-component plasmas in which there are no atoms or molecules. Similar comments appear in a paper by Maggs and Ashcroft.[11]

In this paper we exhibit an approximation to the quantum Coulomb plasma that captures the mechanism by which quantum fluctuations destroy the screening. The present paper will motivate conclusions which we will obtain by a complete mathematical analysis of this model to appear shortly.[6] The approximations we present are a possible strategy by which the conclusions of Alastuey and Martin could be established nonperturbatively, but this appears to be an unreasonably lengthy enterprise at the moment.

To motivate the choice of our model we first review some aspects of the proof of screening in the classical case. We introduce the Gaussian measure $d\mu_{(1/\beta)v_l}(\phi)$ on functions $\phi(x)$ which by definition satisfies

$$\exp\left(-\frac{\beta}{2}\int \rho v_l \rho\right) = \int d\mu_{(1/\beta)v_l}(\phi) \exp\left(-i\beta \int \rho\phi\right) \tag{10}$$

If l were zero, no cutoff, then formally

$$d\mu_{(1/\beta)v_0}(\phi) = D[\phi] \exp\left[-\frac{\beta}{2}\int (\nabla\phi)^2\right] \tag{11}$$

By substituting (10) into the partition function (1) and interchanging the integral over $d\mu$ with the $\sum$ and $\int dp\, d\xi$ we are led to the well known sine-Gordon representation of the partition function. This represents the interacting gas as a superposition over all external fields of ideal-gas partition functions for particles in external fields,

$$Z = \int d\mu_{(1/\beta)v_l}(\phi)\, Z_{\text{ideal}}(i\phi + i\phi_{\text{ext}}) \tag{12}$$

$$Z_{\text{ideal}}(i\phi) = \exp\left[\tilde{z} \int dp\, d\xi\, e^{-\beta h(i\phi)}\right] \tag{13}$$

$$= \exp\left[2ze^{\beta/2l} \int dx \cos \beta\phi(x)\right] \tag{14}$$

where

$$h(i\phi) = \frac{p^2}{2m} + ie\phi \tag{15}$$

It is tempting to make the approximations $\cos \beta\phi \simeq 1 - \frac{1}{2}\beta^2\phi^2$, but this is not quite right in cases where $\beta/l \gg 1$. Instead the first step in the proof of Theorem 1 is to integrate out fluctuations of the field on all scales up to the Debye length l_D. Under the hypotheses of Theorem 1 this is done (exactly) by a Mayer expansion which is convergent because the hypotheses say that the plasma inside a Debye sphere is close to an ideal gas. The result is that the short-distance cutoff l in the Gaussian measure $d\mu_{(1/\beta)v_l}$ in (12) is replaced by l_D while the exponent $2ze^{\beta/2l} \int dx \cos \beta\phi(x)$ becomes a convergent series of nonlocal monomials in $\exp[ie\beta\phi(x)]$, but this series is still dominated by the leading term, which is local and has the form $2z \int dx \cos \beta\phi(x)$. In other words, the effect of a renormalization group transformation is, to a controllable approximation, to replace v_l by

$$v \equiv v_{l_D} \tag{16}$$

in the measure and to drop the constant $e^{\beta/2l}$. In fact there are also renormalizations of parameters, e.g., the dominant term is prefaced by a constant which tends to one as $zl^3e^{\beta/2l} \to 0$, $zl_D^3 \to \infty$, but we shall pretend these are not there throughout this paper.

Choice of Units of Length. Set $l_D = 1$. With this choice of units the hypotheses of Theorem 1 imply that $\beta \ll 1$ and

$$2z = \frac{1}{\beta} \tag{17}$$

Having removed all scales up to $l_D = 1$, the next step in the proof of Theorem 1 is to control the approximation[2]

$$\cos \beta\phi \simeq 1 - \tfrac{1}{2}\beta^2\phi^2 + O(\beta^4) \tag{18}$$

[2] Actually $\exp[(1/\beta)\cos \beta\phi(x)] \approx \exp\{1/\beta\} \sum_n \exp\{-(\beta/2)[\phi - (2\pi/\beta)n]^2\}$.

Within this approximation the partition function becomes, up to constants which cancel in correlations, a Gaussian integral

$$Z \simeq \int d\mu_{(1/\beta)v} \exp\left[-\tfrac{1}{2}\beta \int (\phi + \phi_{\rm ext})^2 \right] \tag{19}$$

$$\equiv \left[\int d\mu_{(1/\beta)v} \exp\left(-\tfrac{1}{2}\beta \int \phi^2 \right) \right] \exp\left(-\tfrac{1}{2}\beta \int \phi_{\rm ext} [1-u]\, \phi_{\rm ext} \right) \tag{20}$$

where u is the *exponentially decaying* kernel of

$$u \equiv (v^{-1} + 1)^{-1} \tag{21}$$

in terms of which one can compute correlations of charge observables by functional derivatives with respect to $\phi_{\rm ext}$ and obtain the results of Debye–Hückel theory, in particular, exponential decay of correlations.

3. THE QUANTUM COULOMB GAS

Now we turn to the analogous representation in the quantum case. For simplicity we discuss the case of Boltzmann statistics, but there are similar representations for Fermi and Bose statistics. This simplification is reasonable since we are discussing a regime in which the gas is very far from degenerate, $l \ll$ interparticle distance by the hypotheses of the theorem.

The correct way to take into account the failure of commutativity $[p, x] \neq 0$ is to replace $\int dp\, d\xi$ by the trace over the one-particle Hilbert space and use time-ordered exponentials in $Z_{\rm ideal}(i\phi)$, so that

$$Z_{\rm ideal}(i\phi) = \exp\left\{ \tilde{z}\, {\rm Tr} \left(\exp\left[-\int_0^\beta d\tau\, h(i\phi) \right] \right) \right\} \tag{22}$$

where ϕ is now an imaginary-time-dependent external field $\phi(\tau, x)$ which is integrated over using the Gaussian measure $d\mu_{v_l \otimes I}$ whose covariance is $v_l(x-y)\, \delta(\tau - \sigma)$. With these substitutions in (12) the sine-Gordon representation (12) is still valid.

To understand this, set $\phi_{\rm ext} = 0$, and consider

$$\sum \frac{\tilde{z}^N}{N!} {\rm Tr}_N\, e^{-\beta H}$$

where H is the many-body quantum Hamiltonian obtained from (2) by $p \to (\hbar/i)\, \partial/\partial x$ and ${\rm Tr}_N$ is the N-body trace. Then

$${\rm Tr}_N \exp(-\beta H) = {\rm Tr}_N \lim_{n \to \infty} \prod_1^n \left[\exp\left(-\frac{\beta}{n} H_0 \right) \exp\left(-\frac{\beta}{2n} \int \rho v_l \rho \right) \right] \tag{23}$$

We use the representation (10) for each factor of $\exp[-(\beta/2n)\int \rho v_l \rho]$, each requiring its own auxiliary field $\phi_i(x)$, $i=1,\ldots, n$. Then

$$\operatorname{Tr}_N \exp(-\beta H) = \lim_{n\to\infty} \int d^n\mu_{(n/\beta)v_l}(\phi)\, \operatorname{Tr}_N \times \prod_1^n \left[\exp\left(-\frac{\beta}{n} H_0\right)\exp\left(-\frac{\beta}{n} i\int \rho\phi_j\right)\right] \tag{24}$$

$$= \lim_{n\to\infty} \int d^n\mu_{(n/\beta)v_l}(\phi)\, \operatorname{Tr}_N \times \exp\left(-\int_0^\beta d\tau \left[H_0 + i\int \rho\phi\right]\right) \tag{25}$$

where the exponential is time ordered and the collection of fields $\phi_j(x)$ is united into one time-dependent field $\phi(\tau, x) \equiv \phi_j(x)$ when

$$\tau \in \left[\frac{(j-1)\beta}{n}, \frac{j\beta}{n}\right)$$

Finally we note that the trace over the many-body Hilbert space factors into a product of one-body traces so that

$$\sum \frac{\tilde{z}^N}{N!} \operatorname{Tr}_N e^{-\beta H} = \int d\mu_{v_l\otimes I}(\phi)\, Z_{\text{ideal}}(i\phi + i\phi_{\text{ext}}) \tag{26}$$

where we have put the external field back in. We write $\int d\mu_{v_l\otimes I}$, but we mean $\lim_{n\to\infty} \int d^n\mu(\phi)$.

Following ref. 8, we can write $Z_{\text{ideal}}(i\phi)$ as a sum over all continuous closed paths $X(\tau)$, $\tau\in[0, \beta]$, using the Feynman–Kac formula,

$$\operatorname{Tr}\left(\exp\left[-\int_0^\beta d\tau\, h(i\phi)\right]\right) = \sum_e \int dW^\beta(X) \exp\left[-ie\int_0^\beta d\tau\, \phi(\tau, X(\tau))\right] \tag{27}$$

dW is the Wiener measure associated with the kernel of $\exp[t((\hbar^2/2m)\Delta)]$. The combination of (12) and (27) is a representation for the quantum partition function which appears in ref. 7. It is also derived and used in ref. 2.

Notice that there is a Goldstone mode:

$$\phi(\tau, x) \to \phi(\tau, x) + f(\tau)$$

where f is any function such that $\int_0^\beta d\tau\, f(\tau) = 0$. This will be the origin of the long-range forces. The intuition is that the Feynman–Kac formula represents the quantum gas as a classical gas of closed charge loops with instantaneous Coulomb interactions. Each loop represents the quantum uncertainty around a classical position. This leads to a time-dependent dipole force superimposed on the Coulomb force for the classical system. A dipole can polarize other dipoles, leading to induced dipole–dipole or multipole–multipole forces which are power laws. The standard textbook discussions do not see this effect because they make a static approximation which loses these time dependences. The mechanism is very similar to the van der Waals forces, except that it takes place without any need for neutral objects such as atoms or molecules.

To bypass some terrible technicalities we now alter the Wiener measure to obtain a simple model which exhibits destruction of screening by the same mechanism that we claim will occur in the complete model.

4. THE SEMIQUANTUM SIMPLIFICATION

We replace the integration $\int dW^\beta$ over all Wiener paths by a new integration concentrated on just one kind of path which oscillates about the initial point by a distance $O(l)$ (the size of the wavepacket) in a random direction: let $d\sigma(\mathbf{e})$ be a spherically symmetric measure on vectors $\mathbf{e}$. Then

$$dW^\beta \to dx\, d\sigma(\mathbf{e}) \tag{28}$$

The right-hand side is a measure on paths because $(x, \mathbf{e})$ labels the path:

$$\begin{aligned} X(\tau) &= x + l\mathbf{e} f(\tau) \\ f(\tau) &= \sin\left[\frac{2\pi\tau}{\beta}\right] \end{aligned} \tag{29}$$

We do not claim that this is a controllable approximation in the sense that there is a physically natural parameter that can be driven to some limit to obtain it, but it is one of the simplest ways to put a little quantum mechanics into a classical model. We shall choose

$$d\sigma(\mathbf{e}) = \frac{1}{2}\left[\delta(\mathbf{e}) + (2\pi)^{-3/2} \exp\left(-\frac{\|\mathbf{e}\|^2}{2}\right)\right] \tag{30}$$

This choice perhaps would look more natural if there were no delta function: the delta function has the interpretation that half our particles are truly classical while the other half are semiquantum. The choice of proportions is not essential; indeed one could allow all the particles to be semiquantum, but the resulting model is harder to analyze rigorously. We now make some more changes, but these, we claim, are on a different footing from the last change. They are attempts to extract an effective Lagrangian, which, we believe, can be justified by rigorous mathematics.

Approximation 1:

$$\int_0^\beta d\tau\,\phi(\tau, X(\tau)) = \int_0^\beta d\tau\,\phi(\tau, X(0)) + \int_0^\beta d\tau \int_{t=0}^{t=\tau} \nabla\phi(\tau, X(t))\cdot dX(t) \tag{31}$$

$$= \phi([0,\beta], x) + \int_{t=0}^{t=\beta} \nabla\phi([t,\beta], X(t))\cdot dX(t)$$

$$\approx \phi([0,\beta], x) + \int_{t=0}^{t=\beta} \nabla\phi([t,\beta], x)\cdot dX(t) \tag{32}$$

where ∇ acts on the spatial variables and

$$\phi([t,\beta], x) \equiv \int_t^\beta d\tau\,\phi(\tau, x) \tag{33}$$

The consequence of these approximations is that the dependence of $Z_{\text{ideal}}(i\phi + i\phi_{\text{ext}})$ on $\phi(\tau, x)$ is only through

$$\phi_1(x) \equiv \frac{1}{\sqrt{\beta}}\,\phi([0,\beta], x)$$

$$\phi_2(x) \equiv \left(\frac{2}{\beta}\right)^{1/2} \int_0^\beta d\tau\,\phi(\tau, x)\,f(\tau)$$

In fact, in terms of these fields we find by the integration by parts

$$\int_{t=0}^{t=\beta} \nabla\phi([t,\beta], x)\,dX(t) = \int_0^\beta dt\,\nabla\phi(t, x)\cdot[X(t) - x]$$

that

$$\int_0^\beta d\tau\,\phi(\tau, X(\tau)) \approx \beta^{1/2}\phi_1(x) + \left(\frac{\beta}{2}\right)^{1/2} l\mathbf{e}\cdot\nabla\phi_2(x) \tag{34}$$

Since the fields ϕ_1, ϕ_2 are Gaussian and $\int d\mu_{v_l \otimes I}\,\phi_i(x)\,\phi_j(y) = v_l(x-y)\,\delta_{ij}$, then ϕ_1, ϕ_2 are independently distributed according to the massless Gaussian measure $d\mu_{v_l}$ encountered above in the classical model.

Thus the partition function becomes

$$Z = \int d\mu_{v_l}(\phi_2) \int d\mu_{v_l}(\phi_1) \cdot \exp\left\{2\tilde{z} \int dx \int d\sigma(\mathbf{e}) \times \cos\left[\beta^{1/2}\phi_1(x) + \beta^{1/2}\phi_{\text{ext}} + \left(\frac{\beta}{2}\right)^{1/2} l\mathbf{e}\cdot\nabla\phi_2\right]\right\} \tag{35}$$

Notice that if $d\sigma(\mathbf{e})$ is set to $\delta(\mathbf{e})$ we revert to the classical Coulomb gas. If ϕ_1 is set to zero, then by reversing the sine-Gordon transformation we obtain the partition function of a classical dipole gas with dipole moments $\mathbf{e}$ distributed according to $d\sigma$.

Approximation 2. This is the same step as discussed above for the classical model in which the fluctuations on scales up to $l_D = 1$ are integrated out by a Mayer expansion, of which we keep only the leading term. Thus v_l becomes $v_1 \equiv v$ and $2\tilde{z}$ becomes $1/\beta$. We have

$$Z \approx \int d\mu_v(\phi_2) \int d\mu_v(\phi_1) \cdot \exp\left\{\frac{1}{\beta} \int dx \int d\sigma(\mathbf{e}) \times \cos\left[\beta^{1/2}\phi_1(x) + \beta^{1/2}\phi_{\text{ext}} + \left(\frac{\beta}{2}\right)^{1/2} \mathbf{e}\cdot l\nabla\phi_2\right]\right\} \tag{36}$$

The integration over $d\sigma(\mathbf{e})$ can be performed explicitly and the partition function becomes

$$Z = \int d\mu_v(\phi_2) \int d\mu_v(\phi_1) \times \exp\left\{\frac{1}{\beta} \int dx\, w_2 \cos[\sqrt{\beta}\,\phi_1(x) + \sqrt{\beta}\,\phi_{\text{ext}}]\right\} \tag{37}$$

$$w_2(x) \equiv \frac{1}{2}\left(1 + \exp\left\{-\frac{1}{4}\beta[l\nabla\phi_2(x)]^2\right\}\right)$$

The next approximation is of the same nature as the quadratic approximation of the cosine used to prove Theorem 1. We have by the hypotheses of Theorem 1 that $\beta \ll 1$, so that we use the following result:

Approximation 3:

$$\frac{1}{\beta}\cos(\sqrt{\beta}\,\phi_1 + \sqrt{\beta}\,\phi_{\text{ext}}) \simeq \frac{1}{\beta} - \frac{1}{2}(\phi_1 + \phi_{\text{ext}})^2$$

Now we can integrate out the ϕ_1 field: let

$$u_{w_2}(x, y) \equiv \text{kernel of the operator } [v^{-1} + w_2]^{-1} \tag{38}$$

Then

$$\begin{aligned}&\int d\mu_v(\phi_1) \exp\left[-\tfrac{1}{2}\int dx\, w_2(\phi_1 + \phi_{\text{ext}})^2\right]\\ &\quad = \left[\int d\mu_v(\phi_1) \exp\left(-\tfrac{1}{2}\int dx\, w_2 \phi_1^2\right)\right]\\ &\qquad \times \exp\left(-\tfrac{1}{2}\int \phi_{\text{ext}}[w_2 - u_{w_2}]\, \phi_{\text{ext}}\right)\end{aligned} \tag{39}$$

5. CONCLUSIONS

These approximations have led us to the following model of the quantum Coulomb gas:

$$\begin{aligned}Z = &\int d\mu_v(\phi_2) \exp\left(\frac{1}{\beta}\int dx\, w_2\right)\left[\int d\mu_v(\phi_1) \exp\left(-\frac{1}{2}\int dx\, w_2 \phi_1^2\right)\right]\\ &\times \exp\left(-\frac{1}{2}\int \phi_{\text{ext}}[w_2 - u_{w_2}]\, \phi_{\text{ext}}\right)\end{aligned} \tag{40}$$

We will give a complete nonperturbative analysis of this model in a forthcoming paper.[6] Let expectations of static charge densities be obtained by functional derivatives

$$\left\langle \prod_{i=1}^{r} \rho(x_i)\right\rangle \equiv \left[\frac{1}{Z}\prod_{i=1}^{r} \frac{\delta}{\delta\phi_{\text{ext}}(x_i)}\, Z(\phi_{\text{ext}})\right]_{\phi_{\text{ext}}=0} \tag{41}$$

Here are the conclusions we expect:

1. The two-point correlations decay exponentially:

$$\langle \rho(x_1)\, \rho(x_2)\rangle \sim \frac{\text{const}}{\beta}\, u(x_1 - x_2) \tag{42}$$

as $|x_1 - x_2| \to \infty$. u is given in (21).

2. The higher correlations do not decay exponentially; for example,

$$\begin{aligned}&\langle \rho(x)\, \rho(-x)\, \rho(x+y)\, \rho(-x+y)\rangle\\ &\quad - \langle \rho(x)\, \rho(-x)\rangle \langle \rho(x+y)\, \rho(-x+y)\rangle\\ &\quad \sim \text{const}\cdot\left[\frac{1}{\beta} u(-x)\, u(x)\right]^2 [\beta l^2]^2 \frac{1}{|y|^6}\end{aligned} \tag{43}$$

as $y \to \infty$ with $|x|$ large.

Justification. Notice that in Eq. (40) the kernel $u_{w_2}(x, y)$ decays uniformly in w_2 because $w_2(x)$ is smooth in x and $w_2(x) \geqslant 1/2$ for all[3] ϕ_2. Therefore functional derivatives with respect to ϕ_{ext} are linked in pairs by exponentially decaying propagators u_{w_2}. However, the propagators u_{w_2} depend on the field $\nabla\phi_2$ through w_2. This field is distributed according to a massless Gaussian measure $d\mu_v$ with a small perturbation by the terms

$$\left[\int d\mu_v(\phi_1)\exp\left(-\frac{1}{2}\int dx\, w_2\phi_1^2\right)\right]$$

$$\approx \exp\left\{\int \left[O(1) + O(1)\,\beta(l\nabla\phi_2(x))^2\right] dx\right\} \tag{44}$$

$$\exp\left(\frac{1}{\beta}\int dx\, w_2\right) \approx \exp\left\{\int\left[\frac{1}{\beta} + O(1)(l\nabla\phi_2)^2\right] dx\right\}$$

Since the perturbation is a function of $\nabla\phi_2$ it will not make the measure massive.

Comments. It is an artifact of this particular approximation and charge symmetry that the action is separately even in ϕ_1 and ϕ_2, which is the reason for the different types of decay. Alastuey and Martin[2] also found that there are different decay rates for the correlations of two versus four charge observables, but the differences were in the exponent of the power law rather than the drastic exponential versus power law that we obtain.

If in Eq. (35) the two fields ϕ_1, ϕ_2 were the same field, then that model would have exponential decay in all correlations. This would be a classical system consisting of particles which carry both a charge and a small dipole moment.

The quadratic approximation

$$\cos\left[\beta^{1/2}\phi_1 + \beta^{1/2}\phi_{\text{ext}} + \left(\frac{\beta}{2}\right)^{1/2} \mathbf{e}\cdot l\nabla\phi_2\right]$$

$$\simeq 1 - \frac{\beta}{2}\left(\phi_1 + \phi_{\text{ext}} + \frac{1}{\sqrt{2}}\mathbf{e}\cdot l\nabla\phi_2\right)^2$$

in Eq. (36) would lose the symmetry $\phi_2 \to \phi_2 + \text{const}$ which is responsible for the destruction of the screening.

[3] This is why we decided to put a δ function into the measure $d\sigma$.

For the full quantum Coulomb partition function (27), Eq. (31) is replaced by

$$\int_0^\beta d\tau\, \phi(\tau, X(\tau)) = \phi([0, \beta], x) + \int_{t=0}^{t=\beta} \nabla\phi([t, \beta], X(t)) \cdot dX(t)$$
$$+ \int_0^\beta \frac{dt}{\beta} l^2 \Delta\phi([t, \beta], X(t)) \qquad (45)$$

The extra term arises by the Ito calculus $dX(t)^2 = (dt/\beta)\, l^2$ and the $dX(t)$ integral is an Ito integral. This formula has the good feature that the very singular field $\phi(\tau, x)$ which is a white noise in its dependence on τ has been traded in for the continuous field $\phi([t, \beta], x)$. It is possible that a nonperturbative proof of the absence of screening for the quantum Coulomb system can be constructed by integrating out the (massive) time-averaged field $(1/\beta)\, \phi([0, \beta], x)$ as we did (approximately) in obtaining the model (40).

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